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REVIEW

Advances in the Physics of High-Temperature Superconductivity

J. Orenstein[1] and A. J. Millis[2]

The high-temperature copper oxide superconductors are of fundamental and enduring interest. They not only manifest superconducting transition temperatures inconceivable 15 years ago, but also exhibit many other properties apparently incompatible with conventional metal physics. The materials expand our notions of what is possible, and compel us to develop new experimental techniques and theoretical concepts. This article provides a perspective on recent developments and their implications for our understanding of interacting electrons in metals.

In a paper published in *Science* very shortly after the 1986 discovery of high–critical temperature (T_c) superconductivity by Bednorz and Müller, Anderson identified three essential features of the new superconductors (*1*). First, the materials are quasi–two-dimensional (2D); the key structural unit is the CuO_2 plane (Fig. 1), and the interplane coupling is very weak. Second, high-T_c superconductivity is created by doping (adding charge carriers to) a "Mott" insulator. Third, and most crucially, Anderson proposed that the combination of proximity to a Mott insulating phase and low dimensionality would cause the doped material to exhibit fundamentally new behavior, not explicable in terms of conventional metal physics.

In the ensuing years this prediction of new physics was confirmed, often in surprising ways. The challenge has become to characterize the new phenomena and to develop the concepts required to understand them. The past 5 years have been particularly exciting. Advances in crystal chemistry and in experimental techniques have created a wealth of information with remarkable implications for high-T_c and related materials. Here we focus on four areas where progress has been especially rapid: spin and charge inhomogeneities ("stripes"); the low-temperature properties of the superconducting state; phase coherence and the origin of the pseudogap; and the Fermi surface and its anisotropies in the nonsuperconducting or normal state.

Mott Insulators, Superconductivity, and Stripes

High-T_c superconductivity is found in copper oxide–based compounds with a variety of crystal structures, an example of which is shown in Fig. 1A. The key element shared by all such structures is the CuO_2 plane, depicted with an occupancy of one electron per unit cell in Fig. 1B. At this electron concentration the plane is a "Mott insulator," the parent state from which high-T_c superconductors are derived. A Mott insulator is a material in which the conductivity vanishes as temperature tends to zero, even though band theory would predict it to be metallic. Many examples are known, including NiO, $LaTiO_3$, and V_2O_3. [For recent reviews, see (*2, 3*).] However, the high-T_c cuprates are the only Mott insulators known to become superconducting when the electron concentration is changed from one per cell.

A Mott insulator is fundamentally different from a conventional (band) insulator. In the latter system, conductivity is blocked by the Pauli exclusion principle. When the highest occupied band contains two electrons per unit cell, electrons cannot move because all orbitals are filled. In a Mott insulator, charge conduction is blocked instead by electron-electron repulsion. When the highest occupied band contains one electron per unit cell, electron motion requires creation of a doubly occupied site. If the electron-electron repulsion is strong enough, this motion is blocked. The amount of charge per cell becomes fixed, leaving only the electron spin on each site to fluctuate. Doping restores electrical conductivity by creating sites to which electrons can jump without incurring a cost in Coulomb repulsion energy.

Virtual charge fluctuations in a Mott insulator generate a "super-exchange" (*4*) interaction, which favors antiparallel alignment of neighboring spins. In many materials, this leads to long-range antiferromagnetic order, as shown in Fig. 1. Anderson proposed that the quantum fluctuations of a 2D spin ½ system like the parent compound La_2CuO_4 might be sufficient to destroy long-range spin order. The resulting "spin liquid" would contain electron pairs whose spins are locked in an antiparallel or "singlet" configuration. The motion of such singlet pairs is akin to the resonance of π bonds in benzene, thus the term "resonating valence bond" (RVB). Anderson pointed out that the valence bonds resemble the Cooper pairs of Bardeen-Cooper-Schrieffer (BCS) superconductivity. A compelling picture of a Mott insulator as a suppressed version of the BCS state emerged: electrons dressed up in pairs, but with no place to go. Because the Mott insulator is naturally paired, Anderson argued, it would become superconducting if the average occupancy is lowered from one.

Soon after the discovery of high-T_c superconductivity, experiments revealed that the spin liquid state is not realized in the undoped cuprates. [It now seems likely that a spin liquid ground state exists for spin ½ particles on geometrically frustrated 2D lattices such as the Kagome (*5*).] Instead, the spins order in a commensurate antiferromagnetic pattern at a rather high Néel temperature between 250 and 400 K, depending on the material. The extent of the antiferromagnetic phase in the temperature versus carrier concentration plane of the high-T_c phase diagram is illustrated in Fig. 2. The Néel temperature drops very rapidly as the average occupancy is reduced from 1 to $1 - x$, reaching zero at a critical doping x_c of only 0.02 in the

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$La_{(2-x)}Sr_xCuO_4$ system, for example.

For doping levels above x_c, various forms of local or incommensurate magnetism survive the loss of commensurate antiferromagnetic order. At intermediate levels, the dynamical properties are those of a spin glass. As the doping increases, the degree of local magnetic order appears to depend on the material system and on sample purity. At doping levels greater than optimal for T_c, magnetic correlations finally become negligible. The extraordinary persistence of antiferromagnetism into the superconducting phase has been a longstanding puzzle. However, in the last few years a startling new picture has emerged. The revelations came largely from neutron scattering results, made possible by the synthesis of large single crystals with controlled values of x. The new phenomenon is inhomogeneous spin and charge ordering, more colloquially known as "stripes."

Neutron scattering in the Néel ordered state at $x = 0$ is relatively simple: The two-sublattice structure (Fig. 1) leads to antiferromagnetic Bragg peaks at wave vectors $\mathbf{Q} = \pm½,\pm½$ (in units of $2\pi/a$, where a is the lattice constant). The location of scattering peaks in momentum space is illustrated in Fig. 3A. The evolution of the spin dynamics with doping was considerably more difficult to understand. Measurements in 1989 and the early 1990s, primarily in $La_{(2-x)}Sr_xCuO_4$, found that broadened versions of the antiferromagnetic Bragg peaks persist in samples with the highest T_c's (*6–9*). Further, the spin correlations were found to be incommensurate: The single peak found in the insulator splits into four (*7–9*), each displaced from $\mathbf{Q}$ by a small amount $\boldsymbol{\delta}$ (Fig. 3A). Many workers had assumed that the spin fluctuations responsible for the scattering could be described by linear response theory and corresponded to a sinusoidal spin density wave (SDW) fluctuating slowly in space and time. However, scattering at $\mathbf{Q} \pm \boldsymbol{\delta}$ can also arise from a spin wave that is locally commensurate but whose phase jumps by π at a periodic array of domain walls termed "antiphase boundaries." Such a spin wave is shown in Fig. 3B.

Tranquada and collaborators found evidence for the latter possibility in a closely related material system in which Nd replaces some of the La atoms in $La_{(2-x)}Sr_xCuO_4$ (*10, 11*). The introduction of Nd changes the direction of a distortion of the crystal structure (arising from slight rotation of the CuO_6 octahedra) from diagonal to parallel to the Cu-O bond. This distortion causes the spin fluctuations to condense into a static SDW. The neutron scattering spectrum of the SDW consists of Bragg peaks that could be analyzed in detail; the analysis confirms that the ordered spin structure is the one shown in Fig. 3B. In this structure the vacant sites introduced by doping reside at antiphase boundaries, forming charged stripes. If there is one vacancy for every two sites along the stripe, then the distance between stripes is $a/2x$ (where a is the Cu-Cu separation). This distance gives the charge periodicity; the antiphase property means that the spin period is twice this value, or a/x. This spin periodicity causes Bragg peaks displaced from $\mathbf{Q}$ by $\delta = x$; the charge periodicity leads to Bragg peaks displaced from fundamental lattice reflections by $2x$. Both features were observed by Tranquada *et al.*, and thus charged stripes, the "latest installment in the mystery series entitled 'High-T_c Superconductivity' " (*12*), were discovered.

The equality of incommensurability and doping, $\delta = x$, follows naturally from the quarter-filled stripe model and has not been explained in any other way. The work of Tranquada *et al.* stimulated a closer investigation of the inelastic peaks in compounds in which the SDW is not static. In 1998 Yamada *et al.* reported that $\delta = x$ in underdoped Nd-free $La_{(2-x)}Sr_xCu_2O_4$, as was found for the spin Bragg peaks in the Nd-doped system (*13*). (This relation is deduced from the spin peak; the charge peak is too weak to see in inelastic scattering measurements.) This strongly suggests that the inelastic incommen-

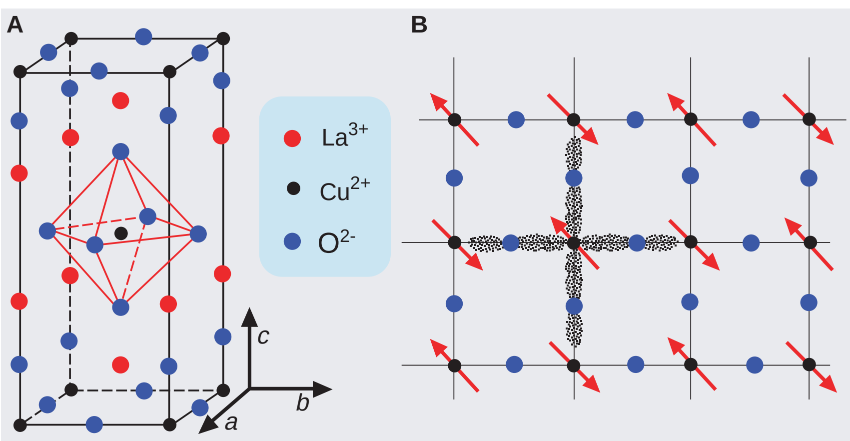

Fig. 1. (**A**) Crystal structure of La_2CuO_4, the "parent compound" of the $La_{(2-x)}Sr_xCuO_4$ family of high-temperature superconductors. The crucial structural subunit is the $Cu\text{-}O_2$ plane, which extends in the *a-b* direction; parts of three CuO_2 planes are shown. Electronic couplings in the interplane (*c*) direction are very weak. In the La_2CuO_4 family of materials, doping is achieved by substituting Sr ions for some of the La ions indicated, or by adding interstitial oxygen. In other families of high-T_c materials (e.g., $YBa_2Cu_3O_{6+x}$) the crystal structure and mechanism of doping are slightly different, but all materials share the feature of CuO_2 planes weakly coupled in the transverse direction. (**B**) Schematic of CuO_2 plane, the crucial structural subunit for high-T_c superconductivity. Red arrows indicate a possible alignment of spins in the antiferromagnetic ground state of La_2CuO_4. Speckled shading indicates oxygen "p_σ orbitals"; coupling through these orbitals leads to superexchange in the insulator and carrier motion in the doped, metallic state.

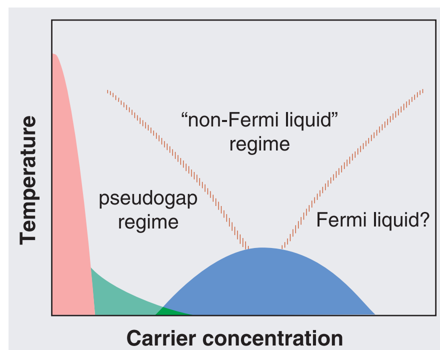

Fig. 2. Schematic phase diagram of high-temperature superconductors. The shaded red area indicates the region in which long-range commensurate antiferromagnetic order (of the type shown by the red arrows in Fig. 1) occurs. The shaded blue area indicates the region in which superconducting long-range order occurs. The carrier concentration at which the superconducting transition temperature (upper boundary of blue region) is maximal is conventionally defined as optimal doping. Materials with lower and higher carrier concentrations are referred to as underdoped and overdoped, respectively. In the regime between the commensurate antiferromagnetic phase and the superconducting phase, a different, more complicated magnetic order occurs; this is discussed more fully in the text, and is shown here as the green region. This order is observed to coexist with superconductivity in some materials, but whether the two phases exist in spatially distinct regions of a sample or coexist in the same region is still not fully clear. The shaded lines indicate qualitatively defined crossover temperatures below which materials, although still thermodynamically in the normal phase, exhibit new behaviors discussed in more detail in the text. (This figure shows the phase-diagram obtained by "hole doping." A few electron-doped materials have been made; because of sample preparation difficulties, their properties are less well determined than those of the hole-doped materials.)

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surate scattering is due to a fluctuating version of the stripes that are static in the Nd-doped samples.

Stripes are an example of a fundamentally new excitation in electronic systems. They are a local and nonlinear phenomenon whose formation is not naturally describable in terms of Fermi liquid theory. The theory of stripes began in 1989 with predictions that were based on mean field approaches (*14–16*). Although these works correctly suggested that stripe structures could occur, they predicted empty, not quarter-filled, stripes. At present, "stripology" is a new and wide-open field. The factors that determine the charge density and direction of stripes in a crystal, as well as the charge mobility along and transverse to the stripes, are questions that are currently under investigation (*17, 18*). This much is generally accepted: The basic physics underlying stripe formation is that of the expulsion of holes from regions of well-formed local moments. In one scenario, the holes would like to separate completely from the spins (*19, 20*). However, the long-range Coulomb repulsion frustrates this desire, and stripes appear as a compromise. In another view (*21, 22*), short-range interactions can lead to stripe formation. The driving force appears to be the lowering of the kinetic energy of holes because of transverse motion of the stripe. It is well known that the hopping of an isolated hole leaves a line of misaligned spins in its wake. However, if the vacancies are confined to an antiphase boundary, the transverse wandering of the stripe does not upset the spins at all.

Beyond stripology, the key question is the role of stripes in superconductivity (*17*). They may be crucial, beneficial, or harmful; all appear as possibilities at the time of this writing. In this regard, there is a key distinction between static and fluctuating stripes. There is evidence that static stripes, or some form of local magnetic order, can exist in superconducting samples (*10, 11, 23, 24*). However, it generally appears that static stripes are antipathetic to superconductivity. The clearest example occurs when $x = 1/8$, where commensurability stabilizes the largest amplitude static stripes in the $La_{(1.6-x)}Nd_{0.4}Sr_xCuO_4$ system (*10, 11, 25*). This doping corresponds to a local minimum in the curve of $T_c(x)$ (*26*). Finally, there is evidence (*25*) that, although static spin order is harmful for superconductivity, static charge order may not be.

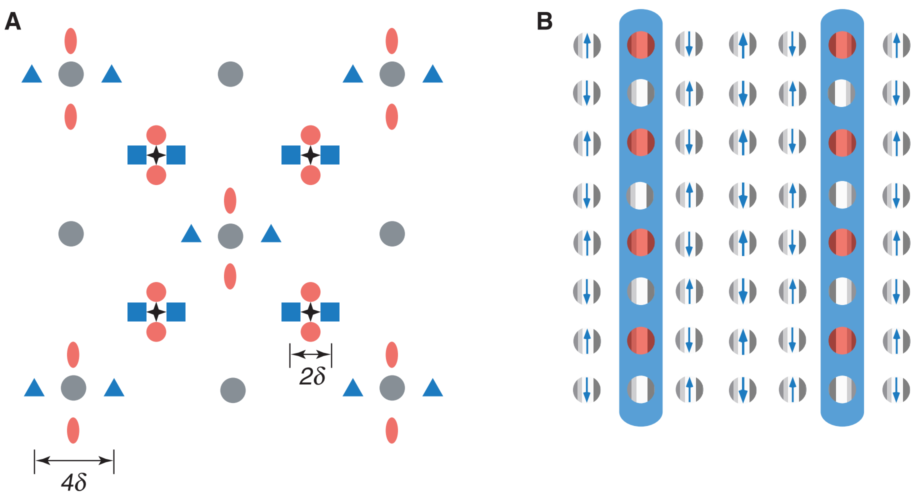

Fig. 3. Neutron scattering: reciprocal and real space. (**A**) Sketch [adapted from (*11*)] of reciprocal (momentum) space for ideal square CuO_2 lattice of lattice constant (Cu-Cu distance) *a*. Large gray dots: fundamental lattice Bragg peaks at wave vectors $\mathbf{Q} = 2\pi/a(m,n)$. Stars: commensurate antiferromagnetic Bragg peaks caused by antiferromagnetic ordering in ideal undoped material. Upon doping, the commensurate antiferromagnetic Bragg peaks disappear and are replaced by four broadened incommensurate dynamic peaks, indicating spin fluctuations displaced from the commensurate peak by a small amount δ related to the doping *x* by $\delta = x$, for $x < x_{opt}$. These are shown as blue squares and red circles. It is now believed that in many cases the underlying spin correlations exhibit a 1D ("stripe") modulation, so that in an ideal monodomain sample either the blue peaks or the red peaks would be observed. In the samples actually studied, four peaks are observed because the stripes run in one direction in some regions and perpendicular to that direction in others. In the $La_{(2-x-y)}Nd_ySr_xCuO_4$ system, long-range order occurs (the peaks become Bragg peaks), and new peaks (shown as blue triangles and red ovals) are observed displaced by 2δ from the fundamental lattice peaks. These are interpreted as the result of charge ordering. (**B**) Schematic illustration of "stripe" ordering, which could give rise to the diffraction pattern shown in (A). Charge is largely confined to the channels shaded in blue. The average charge density along the stripe of $+e$ per two sites is indicated by alternating red and silver circles. Blue arrows indicate magnitude of the magnetic moment on sites containing spins. The stripe is an antiphase boundary for the antiferromagnetic order; in the absence of the stripe, the first and third column from the left would have the same spin orientation, not an opposite one. Oxygen ions are not shown.

It has been suggested (*27*) that stripes promote superconductivity if they are not too static. Evidence for a link between fluctuating stripes and superconductivity is provided by the "Yamada plot," the remarkable linear relationship between T_c and δ (*13*). Although first documented in the $La_{2-x}Sr_xCuO_4$ system, there is considerable evidence for the same effect in the $YBa_2Cu_3O_{(7-\delta)}$ system (*28, 29*). The Yamada plot suggests that T_c increases if and only if stripes move closer together. A more skeptical view is that T_c and δ increase with *x* (and saturate near optimal doping) for different reasons, and the apparent correlation is accidental. The nature of the connection between superconductivity and charged stripes will likely continue as a very active research area for some time to come.

Superconductivity

We turn now to the low-temperature properties of the superconducting state. Superconductivity is characterized by an order parameter, Ψ, which expresses the way in which the superconductor differs from the normal state, just as magnetization characterizes how a ferromagnet differs from the nonmagnetic state above the Curie temperature. In conventional superconductors, Ψ is a "pair field," a quantum mechanical amplitude for finding two electrons in a paired state. The order parameter is complex, that is, it has both magnitude and phase. The pair field magnitude gives the BCS energy gap Δ. In familiar superconductors such as Pb or Al, Δ is essentially independent of position on the Fermi surface, corresponding to pairs in a rotationally symmetric or s-wave state. In the heavy fermion materials and in ^3He, the pairs may be in p- or d-wave states.

Experiments in the late 1980s established that in high-T_c materials Ψ is a pair field, just as in conventional superconductors (*30*). Early suggestions that it might break time-reversal symmetry have been ruled out experimentally (*31*). In the early 1990s the groups of van Harlingen (*32*) and of Kirtley and Tsuei (*33*) showed that the symmetry of the order parameter is d-wave, that is, it changes sign under a 90° rotation. The sign change means that the gap may vanish at points on the Fermi surface, and angle-resolved photoemission spectroscopy (ARPES) has shown (*34–36*) that $\Delta(\mathbf{k}) \sim \Delta_0[\cos(k_x a) - \cos(k_y a)]$, where $\mathbf{k}$ is the wave vector. This is the $d_{x^2-y^2}$ form of the gap, and is maximal for momenta parallel to the Cu-O-Cu bond and vanishing for momenta at angles of 45° to this bond. The four Fermi surface points at which the gap magnitude vanishes are the nodes.

Now that superconductivity has been established to be d-wave, the next question is whether it can be described by a BCS-like theory, suitably modified to include a d-wave

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gap. Some ideas based on new excitations such as stripes, fractionalized electrons (see below), new symmetries relating superconductivity and magnetism (*37*), or quantum critical points [see (*38*)] suggest a non-BCS state. The testing ground is the low-energy excitation spectrum, as reflected in the low-temperature properties. In the d-wave version of BCS, the only important excitations are nodal quasiparticles, whose energy above the ground state, ϵ, is zero when their momentum coincides with a nodal point. The nodal particles have a Dirac spectrum, that is, $\varepsilon = vp$, where v is a characteristic velocity and p is their momentum relative to the nodal point. The velocity is anisotropic: $v = v_F$ for p perpendicular to the Fermi surface, and $v_\Delta \approx v_F/20$ for motion along the Fermi surface. Thus, the quasiparticle dispersion has the shape of an anisotropic cone with an elliptical cross section. The density of states corresponding to this dispersion is linear in energy, or $g(\varepsilon) \propto \varepsilon/v_\Delta v_F$.

In the last few years, there has been direct confirmation of this nodal quasiparticle spectrum. The initial evidence for a linear density of states was the celebrated result of Hardy, Bonn, and co-workers (*39*) that the low-temperature superfluid density ρ_s decreases linearly with increasing temperature, according to $\rho_s(T) = \rho_s(0) - \alpha T$. Subsequently, both principal velocities of the dispersion cone in $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO) were measured by ARPES (*40*). Recently, it was recognized that α can be renormalized by interactions and does not necessarily probe the bare dispersion measured by ARPES (*41, 42*). Thermal measurements, which probe quasiparticles without dragging along the condensate, are unrenormalized (*42*). The most remarkable example is the thermal conductivity κ, which has been shown theoretically to depend only on the ratio v_Δ/v_F (*43, 44*). The theory has been experimentally verified: Taillefer's group (*45*) showed that their thermal measurements imply values for v_F and v_Δ within 10% of the values found in independent ARPES experiments. This result not only confirms our understanding of the quasiparticle spectrum but also shows that difficulties with materials are being solved, because bulk thermal measurements agree with ARPES, which probes the surface. Ultimately, probes that measure the effect of vortices (*46*) and impurities (*47*) on the quasiparticle density of states, with atomic-scale resolution, may provide the most stringent tests of the $d_{x^2-y^2}$ ground state and the possibility of low-lying states of different order parameter symmetry.

With strong evidence that quasiparticles exist and dominate the thermal properties, the next test for conventional behavior is their lifetime. Here the picture is not quite as clear. Because the Fermi surface shrinks to four nodal points in a d-wave superconductor, the phase space for scattering is severely reduced. Calculations based on BCS theory predict that the quasiparticle lifetime diverges at least as fast as T^{-3} at low T (*48*). Measurements of the microwave absorption in ultrapure YBCO samples, which reveal that the transport lifetime increases as T^{-4} (or perhaps exponentially), appear to confirm this prediction (*49*). Below 20 K, the scattering rate is only 20 GHz (comparable to GaAs superlattices) and the mean free path is ~1 μm. These extraordinarily long mean free paths demonstrate the remarkable sample qualities that have been achieved.

Microwave measurements give the quasiparticle "transport lifetime," the inverse of the rate at which collisions relax a current. The quasiparticle lifetime (the inverse of the rate at which a quasiparticle is scattered out of any given state) is measured directly by ARPES. Indeed, ARPES has revealed, in great detail, the remarkable behavior of the quasiparticle self-energy in the antinodal direction (see below). However, until recently the ARPES lineshape of nodal quasiparticles has been blurred by instrumental resolution. Just in the past year, advances in detector technology have permitted a first look at the ARPES lineshape of nodal quasiparticles in the BSCCO family of high-T_c materials (where superior surface quality is favorable for ARPES experiments).

Valla *et al*. reported measurements of the nodal quasiparticle lineshape from room temperature to $T = 48$ K (*50*). As the temperature is reduced below T_c, they found that the quasiparticle lifetime increases only as T^{-1} instead of the much more rapid increase that is theoretically expected. This is an extension of the well-known normal-state behavior of the quasiparticle lifetime and suggests that nodal quasiparticles are affected only weakly by the onset of superconductivity. It would be truly remarkable if the lifetime is found to increase as T^{-1} in BSCCO as $T \to 0$. From a theoretical viewpoint, the persistence of a T-linear scattering rate to very low T would argue against a placid BCS-like ground state. It would require that quasiparticles feel the presence of strong fluctuations at low energy, of the kind expected near a quantum critical point (*38*). [A large literature exists on approaches to quasiparticle transport based on quantum criticality in high-T_c materials. For some examples, see (*51–55*).]

Although some aspects of these photoemission results are currently controversial (*56*), the possibility that nodal quasiparticles are much more strongly scattered in BSCCO than in YBCO is supported by thermal (*57*) and ac electrical (*58–60*) conductivity measurements. While it is generally agreed that BSCCO samples have not reached the degree of purity and structural perfection obtained in the YBCO system, this is not likely to account for the differences at high temperatures where the dominant scattering is inelastic. Given the fact that YBCO and BSCCO have the same CuO_2 bilayer structure and nearly identical values of T_c, the apparent contrast in properties is very puzzling. This problem is currently being addressed actively, both experimentally and theoretically.

Phase Stiffness, Coherence, and Pseudogaps

We now turn to the question of how superconductivity is destroyed either by raising temperature or by changing doping. There is wide agreement on the determining factor for T_c in underdoped materials, largely due to the pioneering experiments of Uemura and collaborators (*61*) and some insightful theory and phenomenology. Uemura *et al*. found that T_c is proportional to the zero-temperature superfluid density (or phase stiffness) $\rho_s(T = 0)$ for a wide range of underdoped materials. At about the same time, it was shown theoretically that this relation was a natural consequence of proximity to a Mott transition (*62, 63*). The implications of the Uemura relation were framed in a more general way by Emery and Kivelson (*64*) in 1995. They pointed out that a conventional superconductor has two important energy scales: the BCS gap Δ, which measures the strength of the binding of electrons into Cooper pairs, and the phase stiffness ρ_s, which measures the ability of the superconducting state to carry a supercurrent. In conventional superconductors, Δ is much smaller than ρ_s and the destruction of superconductivity begins with the breakup of electron pairs. However, in cuprates the two energy scales are more closely balanced; indeed, in underdoped materials the ordering is apparently reversed, with the phase stiffness now the weaker link. When the temperature exceeds $\sim\rho_s$, thermal agitation will destroy the ability of the superconductor to carry a supercurrent while the pairs continue to exist; thus, T_c is bounded above by a pure number times $\rho_s(T = 0)$.

Lee and Wen (*65*) pointed out that nodal quasiparticles can modify this picture in an important way. The thermal excitation of these particles depletes ρ_s because they do not participate in the superflow. In a 2D d-wave superconductor, the expected depletion is linear in the temperature (*66*) (or $d\rho_s/dT = -\alpha$), as found experimentally by Hardy *et al*. (*39*). Although superconductivity is ultimately destroyed by phase fluctuations, quasiparticles play a crucial role in weakening the phase stiffness, allowing phase fluctuations to finish the job. T_c is now controlled not only by the superfluid density at $T = 0$, but by α as well. The Uemura relation for underdoped cuprates, $T_c \propto \rho_s(0)$, survives if α is essentially doping-independent, as indeed was found

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experimentally (*67, 68*). Detailed studies (*68, 69*) have shown that there is a range of carrier concentration above the optimal for superconductivity (x_{opt}) in which ρ_s increases as T_c decreases. Such behavior is consistent with the gap for quasiparticle excitations closing rapidly as carrier concentration becomes larger than x_{opt}.

The explanation for the Uemura relation offered by Lee and Wen relies on the experimental observation that α is doping-independent. However, this observation actually represents a crisis for many theories of superconductivity arising from a doped Mott insulator. A natural prediction of such theories is that the quasiparticle charge renormalizes to zero as the carrier concentration decreases (*70, 71*). This implies that the slope of ρ_s versus T should decrease in samples that are more underdoped, in contrast with experimental results. At present there seem to be two possibilities: Either experiments are not conducted at small enough x to access the asymptotic behavior, or something fundamental is not understood about the approach to the Mott transition in high-T_c materials.

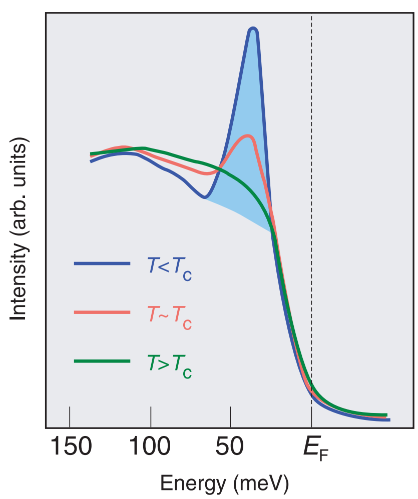

Fig. 4. Representation of ARPES spectra [adapted from (*72, 73*)] for underdoped high-T_c superconductor at momentum near the $(0,\pi)$ or "antinodal" point of the Brillouin zone. Shown is photoemission intensity (proportional to the probability of finding an electron at the given momentum and energy) at fixed momentum, as a function of energy measured relative to E_F, at three different temperatures. At low temperatures, the spectrum shows the behavior expected for a d-wave superconductor at a momentum for which the gap is large. The superconducting gap is seen as a suppression of intensity at low energies. The sharp peak shaded in blue is interpreted as a "quasiparticle" or well-defined electronic excitation. As the temperature is raised through T_c, the quasiparticle peak vanishes but the gap persists. In an overdoped material (not shown), the gap would collapse as T was increased through T_c, but a quasiparticle peak would be visible at all values of T (albeit broadened at $T > T_c$).

Upon crossing the superconducting phase boundary into the normal state, one enters a fascinating and subtle region of the phase diagram. Figure 4 shows schematically how the ARPES spectra of the "antinodal" region of momentum space evolve upon crossing this boundary. [For more details on the evolution of the spectra through the transition, see (*72, 73*). For reviews of ARPES in high-T_c superconductors, see (*74, 75*).] In a conventional metal, the spectrum would contain a narrow peak, which is the signature of a well-defined quasiparticle. Below T_c, exactly such a peak is seen. The leading edge is also set back from the Fermi level, indicating the presence of a gap. As Fig. 4 shows, the quasiparticle peak disappears upon warming through T_c. Initially, it was thought that this effect had a fairly conventional explanation based on the closing of the superconducting gap. Basically, broadening reflects the scattering of electrons off some other degree of freedom. If the gap disappears, the phase space available for scattering is increased, leading to broadening of the quasiparticle peak. The data indicate precisely the opposite effect. Upon crossing into the normal state, the gap (as seen from the leading edge relative to the Fermi energy E_F) is preserved, yet the narrow peak is gone. Evidently, the quasiparticle owes its existence to the phase coherence of the superconducting state, and not the energy gap.

This behavior is very difficult to understand in a phase transition from a superconductor to a Fermi liquid. The data are more naturally understood if electrons in the normal state fractionalize into separate spin- and charge-carrying entities (*1, 70, 76, 77*). In this view, the normal-state spectra are broad because an electron rapidly decays into its more fundamental constituents. In RVB and related models, the narrowing upon cooling below T_c is a consequence of condensation of the charged particles; in stripe models, this narrowing is the result of a dimensional crossover from 1D to 2D. Spin-charge separation is known to occur in one dimension; when, and if, it occurs in two dimensions is a subject of current controversy.

The behavior upon warming further depends very strongly on doping (see Fig. 2). The most interesting properties are seen on the underdoped side of the phase diagram. According to the phase fluctuation picture described earlier, T_c marks the destruction of infinite-range phase order. Above T_c one expects a regime in which the phase remains coherent in finite, but nonzero, intervals of length and time. The signature of such partial phase coherence is that the phase stiffness becomes frequency-dependent above T_c (*78*). At very low frequency ρ_s will be zero, as expected for a normal material. However, if ρ_s were measured at a frequency greater than the dephasing rate, the superfluid density would tend to a nonzero value proportional to the short-range or "bare" phase stiffness.

The frequency dependence of ρ_s can be determined from measurements of the ac conductivity. However, tracking the bare ρ_s well into the normal state requires that the measurement frequency be at least comparable to the maximum dephasing rate, which is ~ $k_B T_c/\hbar$ (where k_B is the Boltzmann constant and $\hbar$ is the Planck constant divided by 2π) or ~1 THz for underdoped materials. Recently, the bare phase stiffness and dephasing rates as a function of T were measured in underdoped BSCCO using a time-domain technique optimized for the terahertz region of the spectrum (*79*). The measurements verified that the transition to the normal state takes place when ρ_s is comparable to T_c. A frequency-dependent ρ_s could be detected in a temperature interval of about 10 to 20 K above the transition. The crossover to the completely incoherent regime takes place when the dephasing rate reaches $k_B T$. Corroborating evidence for a regime of partial coherence is the persistence of the other manifestations of phase coherence above T_c. Aside from the ARPES peak at $(\pi,0)$ mentioned earlier, the "triplet resonance" observed in neutron scattering persists above T_c in underdoped samples (*80*) but broadens rapidly when all traces of phase stiffness are lost.

The most fundamental property of the fully incoherent regime in underdoped materials is the d-wave pseudogap (*74, 75*). The large gap in the antinodal region of momentum space is especially clear. Near the node, where the gap becomes smaller than $k_B T$, the Fermi point appears to be replaced by a small "arc" of Fermi surface. What is striking about this regime is that the gap affects some response functions and is invisible to others. The spin fluctuations are clearly gapped. The charge transport in the CuO_2 planes is largely unaffected (*81*), whereas the transport of charge from one plane to another is strongly suppressed (*82*).

To many, these seemingly bizarre facts are strong evidence for spin-charge separation. Indeed, Kotliar, Fukuyama, and Lee and their collaborators showed that a theoretical implementation of Anderson's RVB ideas led to d-wave pairing, $T_c \sim x$, and a pseudogap regime (*70*). In this picture the pseudogap reflects the singlet pairing of the particles that carry spin. Because they are neutral, this does not affect charge transport in the plane. The behavior of the c-axis transport also fits nicely. The spinless charge carrier is a topological excitation whose realm is strictly 2D, whereas charge transport out of the plane requires the hopping of a real electron. Electricity

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cannot be conducted from one plane to another unless the electron is put back together, at the cost of the spin gap energy (*83*).

As appealing as this scenario is, there is a more down-to-earth alternative. By an accident of band structure, the interplane hopping matrix element vanishes at the nodal point (*84, 85*). If normal-state transport in underdoped cuprates is dominated by the Fermi arc, the *c*-axis conductivity can be strongly suppressed (*86*).

The Normal State

What is usually termed the "normal" state of the cuprates is reached upon crossing the temperature T^* where the pseudogap disappears. The salient features of the strange normal state are as follows. First, there is a connected Fermi surface that appears to be consistent with conventional band theory. Second, there is a striking anisotropy in properties such as ARPES linewidth as one moves around the Fermi surface (*74, 75*). Third, as noted in the early "marginal Fermi liquid" phenomenology (*87*), temperature is the main energy scale governing the spin and charge response functions. A natural explanation of this behavior is that the normal state is a quantum critical regime (*38*). A quantum critical regime implies a quantum critical point separating two $T = 0$ phases. The existence of such a phase transition is suggested by recent transport measurements in which superconductivity was suppressed by the application of very large magnetic fields (*88*). The results suggest that the high-field nonsuperconducting state is insulating for $x < x_{opt}$ and metallic for $x > x_{opt}$.

Supporting a consistent phenomenology on the quantum critical framework remains a challenge. One difficulty is that some of the characteristic T dependences persist up to very high temperatures, on the order of 1000 K in some cases (*89*). Another concerns two celebrated and deceptively simple normal-state properties, the $1/T$ divergence of the conductivity and $1/T^2$ divergence of the Hall angle (*90*). A natural explanation is that charge carriers scatter from singular fluctuations arising from proximity to a quantum phase transition. However, the recent measurements of spin dynamics reported by Aeppli and collaborators (*91*) suggest that singular fluctuations, if they exist, take place at the incommensurate wave vectors described earlier. Hlubina and Rice (*92*) pointed out that only special ("hot") regions of the Fermi surface connected by these wave vectors feel the singular fluctuations. This leaves plenty of "cold spots" that can carry current without singular scattering. Stojkovic and Pines (*52*) have argued that a sum of contributions from cold and hot spots can account for the conductivity and Hall effect power laws. However, the description of the Hall effect is controversial (*93*). Also, there is some reluctance to assign simple power laws, observed in a wide range of temperature, to a balancing of different contributions. Ioffe and Millis (*86*) recently suggested that transport arises entirely from a cold spot, where the scattering rate varies as T^2. This picture is interesting because it does not rest on quantum critical scaling but is consistent with both conductivity and Hall effect power laws. However, it has yet to be derived from a fundamental theory and may be inconsistent with recent ARPES results (*50*).

Anisotropy and Inhomogeneity

It is striking that essentially all phenomenological approaches to high T_c have converged on the theme of strong heterogeneity. In models based on stripes, the solid is viewed as inhomogeneous in real space. In approaches where the system is viewed as spatially homogeneous, there exists a strong heterogeneity in momentum space. The interplay of these ideas is especially apparent when considering the ARPES data on an optimally doped cuprate. The Fermi surface determined by ARPES is shown as the dotted line in Fig. 5 (*94*). It is interesting to compare this contour with the Fermi surface expected for a system containing stripes (*95*). For a single quarter-filled stripe, the occupied states lie in the interval $-\pi/4a < k < \pi/4a$. For an array of horizontal noninteracting stripes embedded in 2D, the filled states lie in the vertical swath shown in the center of the figure. If ARPES were to sample regions with horizontal and vertical stripes, the occupied states would consist of overlapping perpendicular swaths, shown as red and blue areas in the figure. The resulting Fermi surface has a surprising correspondence with experiment (*96*). This is particularly evident in the antinodal regions of the Brillouin zone where this simple idea captures the position of the Fermi surface. However, if one focuses instead on the nodal directions, the simple stripe picture appears to miss some essential physics. In this direction the Fermi surface is bowed so that it is perpendicular to the nodal direction. The Fermi velocity is very large, and the "quasiparticle peak" is well defined and of large amplitude. There is no special significance to the nodal direction in the simple stripe model. The nodal direction is better described in the normal state as a marginal Fermi liquid (*87*) and in the pseudogap regime as a quantum or thermally disordered d-wave superconductor.

The recent and remarkable Hall effect measurements of Uchida and collaborators underscore the question of 1D versus 2D physics in the cuprates (*97*). These experiments probe the impact of stripes on charge transport by studying the $La_{(1.6-x)}Nd_{0.4}Sr_xCuO_4$ system in which spin and charge density order is known to occur. The idea of the experiment is as follows: Even if stripe order is present, the measured conductivity is likely to be isotropic because it averages over regions with different stripe orientations. However, the Hall effect vanishes if the transport is purely 1D, regardless of orientation. Uchida *et al.* showed that the Hall voltage decreases sharply when the sample is cooled below 70 K, the temperature where static stripes first appear. This is direct evidence that the electron motion indeed becomes 1D in the presence of static stripes. However, the Hall effect is not suppressed in Nd-doped samples above 70 K or in Nd-free samples at any temperature. The implications of this measurement for the fluctuating stripe interpretation of high-T_c materials are an area of active research.

Conclusions

The rapid progress of the past 5 years has highlighted the interconnection of materials science and condensed matter physics. The crucial lesson is that searching for new materials and improving the crystal quality of ones already known are the sine qua non of progress. These efforts require a sustained commitment of time, money, and effort. Despite its importance, materials synthesis often has trouble attracting its fair share of glory and funding in a competitive environment.

A second crucial factor contributing to

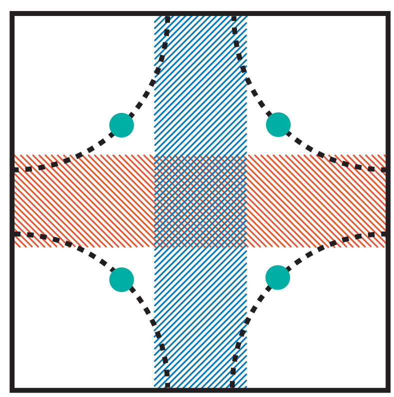

Fig. 5. Representation of Fermi surface observed experimentally (dotted line) and region of filled momentum states predicted by the simple stripe picture [adapted from (*96*)]. The blue regions correspond to stripes that are horizontal in real space; the red regions correspond to vertical stripes. The black box represents the first Brillouin zone of an ideal CuO_2 plane. The four corners correspond to momenta ($\pm\pi/a$, $\pm\pi/a$). The green dots indicate points on the Fermi surface (which become the gap nodes in the superconducting state), where experiments reveal reasonably well-defined quasiparticles propagating with high velocity at 45° to the presumed stripe direction.

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progress has been improved experimental technique. Advances in such methods as microwave, terahertz, and optical spectroscopy, ARPES, scanned-probe microscopy, neutron scattering, and transport in high fields have made available an unprecedented wealth of information, replacing guesswork and speculation with facts.

What has been learned? The important role of inhomogeneity, in real and momentum space, has been recognized. The nature of the antiferromagnetic and superconducting ground states and their low-lying excitations has been revealed in great detail. The agreement between the ARPES and low-T transport properties of d-wave nodal quasiparticles is a particularly impressive triumph of experiment and theory. The findings (at least in YBCO) may be said to support a conventional BCS d-wave spectrum. The pace of progress is reflected in the fact that d-wave superconductivity, regarded as an impossibly exotic and probably irrelevant theoretical speculation only a decade ago, can now be characterized as a manifestation of conventional physics.

A clear qualitative understanding of the "pseudogap" regime has emerged: The gap is due to pairing without long-range order. Superfluid properties are not observed in this regime because the phase stiffness is so small that thermal fluctuations have destroyed the ability of the material to carry a supercurrent. The extent of the pseudogap regime shows that the pairing temperature grows rapidly as doping is decreased, reaching as high as 300 K. This should encourage us not to regard ~150 K as an upper bound for T_c, and to continue to search for materials and mechanisms to achieve superconductivity at room temperature. From a physics point of view, the pseudogap raises the question of whether pairing is a low-energy instability in which quasiparticles are bound into Cooper pairs (as in BCS theory) or is instead a fundamental property of the doped Mott insulating state (as in RVB and related models).

The issue with the broadest implications is the universality of Landau's quasiparticle picture. The low-energy excitations in the superconducting state seem to be the familiar ones, with scattering that is weak or marginal. However, as the temperature is raised, quasiparticles appear to vanish with the loss of superconducting phase coherence. Can these phenomena be explained in terms of conventional quasiparticles, subject to a strong scattering process that depends on the phase ordering transition? Or do we require that the electron split into separate spin- and charge-carrying particles? Is this effect a manifestation of fluctuating stripes, where theories suggest we should abandon the idea of quasiparticles entirely in favor of collective excitations?

The last 5 years of high-T_c research have provided ample evidence that the excitations in a new class of materials are not electron quasiparticles. The next 5 years may tell us, in high-T_c and other materials, what they are. We suspect that the theoretical ideas that emerge will have implications, beyond the area of exotic metals, for the broad area of strongly interacting quantum systems.

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